

Instructions: Time **50** min. Open book and notes exam. No electronics. No questions are allowed. **Bold** case is used for vectors and matrices. Show your work clearly. Every problem is for 6 marks.

Q1. Assume that you have a pattern recognition problem of two classes: **salmon** and **sea bass**. The training sample includes 300 fish with the frequencies shown in the following table. The table shows the fish frequencies according to two qualitative features: **lightness** and **width**. Using Bayes classification rule, find the class of an unknown fish that is light and narrow. *You must show your work clearly.*

		Salmon		Sea Bass	
		Lightness		Lightness	
		Dark	Light	Dark	Light
Width	Narrow	55	5	40	10
	Wide	10	30	20	130
		100		200	
		ω_1		ω_2	

$$P(\omega_1)P(x|\omega_1) <?> P(\omega_2)P(x|\omega_2)$$

$$\frac{100}{300} \times \frac{5}{100} <?> \frac{200}{300} \times \frac{10}{200}$$

$$\frac{5}{300} < \frac{10}{300}$$

Hence it is $\omega_2 \equiv$ Sea Bass

Q2. In a three-class recognition problem, find the value of $-\frac{1}{2}\boldsymbol{\mu}_i^T\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\mu}_i$ given that $\boldsymbol{\Sigma}_i = 4\boldsymbol{I}$ and $\boldsymbol{\mu}_i = [1 \ 2 \ 3]^T$, where \boldsymbol{I} is the identity matrix of size 3-by-3.

$$\boldsymbol{\Sigma}_i = \boldsymbol{\Sigma} = 4\boldsymbol{I}$$

$$\boldsymbol{\Sigma} \boldsymbol{\Sigma}^{-1} = 4\boldsymbol{I} \boldsymbol{\Sigma}^{-1} = \boldsymbol{I}$$

$$\boldsymbol{I} \boldsymbol{\Sigma}^{-1} = \frac{1}{4}\boldsymbol{I}$$

$$\boldsymbol{\Sigma}^{-1} = \frac{1}{4}\boldsymbol{I}$$

$$-\frac{1}{2}\boldsymbol{\mu}_i^T\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\mu}_i = -\frac{1}{2}\boldsymbol{\mu}_i^T\frac{1}{4}\boldsymbol{I}\boldsymbol{\mu}_i = -\frac{1}{8}\boldsymbol{\mu}_i^T\boldsymbol{\mu}_i = -\frac{1}{8}[1 \ 2 \ 3] \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = -\frac{1}{8}(1 + 4 + 8) = -\frac{14}{8} = -\frac{7}{4}$$


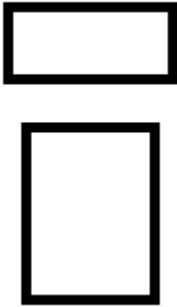

Q3. In a three-class recognition problem, write Matlab code to find the value of $-\frac{1}{2}\boldsymbol{\mu}_i^T \boldsymbol{\Sigma}_i^{-1} \boldsymbol{\mu}_i$ given that $\boldsymbol{\Sigma}_i = 4\mathbf{I}$ and $\boldsymbol{\mu}_i = [1 \ 2 \ 3]^T$, where \mathbf{I} is the identity matrix of size 3-by-3.

```
u = [1 2 3]';
```

```
I = [1 0 0; 0 1 0; 0 0 1];
```

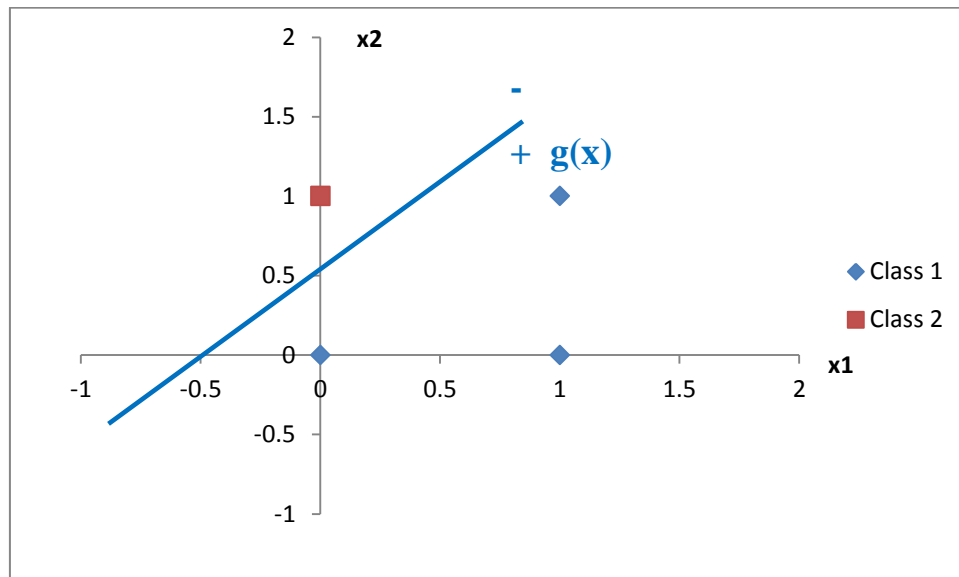
```
-0.5*u'*inv(4*I)u
```

Q4. Suggest a feature extraction technique suitable in classifying three geometric shapes: **circle**, **rectangle**, and **triangle**. The following shows some example samples of this problem. Note that the size and shape can change within any of the three classes.

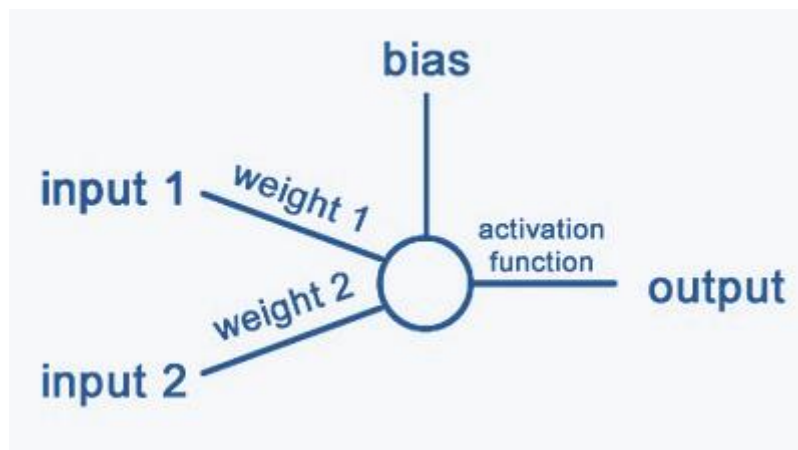
Circle	Rectangle	Triangle
		

- **Fill the inside space**
- **Find the area A**
- **Find the shape width (W) and height (H)**
- **Calculate the feature $A/(W \times H)$**
 - **Circle:** $\pi/4$
 - **Rectangle:** 1
 - **Triangle:** $1/2$

Q5. The following graph shows the feature values of 4 samples (three samples of Class 1 and one sample of Class 2).



a) Draw an appropriate perceptron network that can solve this problem.



b) Find the synaptic weights and thresholds of the neurons of the above network.

$$g(x) = x_1 - x_2 + 0.5$$

Hence $w_1 = 1, w_2 = -1, w_0 = 0.5$

<Good Luck>