0907542 Patter Recognition (Spring 2013)
Midterm Exam

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Instructions: Time **60** min. Open book and notes exam. No electronics. No questions are allowed. **Bold** case is used for vectors and matrices. Show your work clearly. Every problem is for 10 marks.

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Q1. In a two-class recognition problem of normal distributions, $P(\omega_1) = P(\omega_2) = 0.5$, $\boldsymbol{\mu}_1 = [0 \quad 0]^T$, $\boldsymbol{\mu}_2 = [1 \quad 1]^T$, and $\Sigma_1 = \Sigma_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$. Recall that the decision hyperplane function $g_{12}(\boldsymbol{x}) = \mathbf{w}^T(\boldsymbol{x} - \boldsymbol{x}_0)$. Given an unknown sample that has the feature vector $\boldsymbol{x} = [0.8 \quad 0.3]^T$,

a) Classify this unknown sample by evaluating $g_{12}(\boldsymbol{x})$

$$g_{12}(\boldsymbol{x}) = \mathbf{w}^T(\boldsymbol{x} - \boldsymbol{x}_0)$$

$$\boldsymbol{w} = (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$\boldsymbol{x}_0 = \frac{1}{2}(\boldsymbol{\mu}_1 + \boldsymbol{\mu}_2) - \sigma^2 \ln \frac{P(\omega_1)}{P(\omega_2)} \times \frac{\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2}{\| \boldsymbol{\mu}_1 - \boldsymbol{\mu}_2 \|^2}$$

$$\ln \frac{P(\omega_1)}{P(\omega_2)} = 0$$

$$\boldsymbol{x}_0 = \frac{1}{2}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) - 0 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

$$g_{12}(\boldsymbol{x}) = [-1 - 1]\left(\begin{bmatrix} 0.8 \\ 0.3 \end{bmatrix} - \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}\right) = -0.1$$

As $g_{12}(\boldsymbol{x})$ is negative, $\boldsymbol{x}$ belongs to Class ω_2

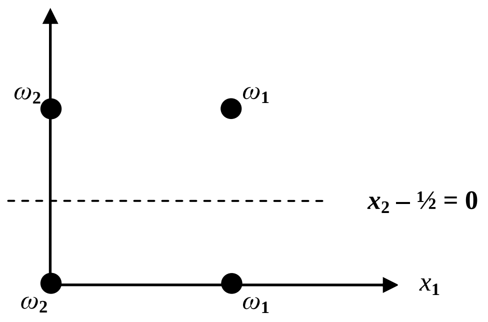
b) Write Matlab code to find $g_{12}(\boldsymbol{x})$

```
m1 = [0 0]';
m2 = [1 1]';
x = [0.8 0.3]';
x0 = 0.5 * (m1 + m2);
w = m1 - m2;
g12 = w' * (x - x0)
```

Q2. It is required to solve a two-class problem using the following version of the perceptron algorithm: $\boldsymbol{w}(t+1) = \boldsymbol{w}(t) - \frac{1}{t+1}\sum_{x\in Y} \delta_x \boldsymbol{x}$. The known samples are two samples of Class ω_1: $[1 \quad 0]^T$ and $[1 \quad 1]^T$, and two samples of Class ω_2: $[0 \quad 0]^T$ and $[0 \quad 1]^T$. Assume that $\boldsymbol{w}(t=0) = \begin{bmatrix}0 & 1 & \frac{-1}{2}\end{bmatrix}^T$.

$$g_{t=0}(\boldsymbol{x}) = \boldsymbol{w}^T(t=0) \times \boldsymbol{x} = \begin{bmatrix}0 & 1 & \frac{-1}{2}\end{bmatrix}\begin{bmatrix}x_1\\x_2\\1\end{bmatrix} = x_2 - \frac{1}{2}$$

The figure to the right shows two miss-classified samples: $[1 \quad 0]^T$ and $[0 \quad 1]^T$



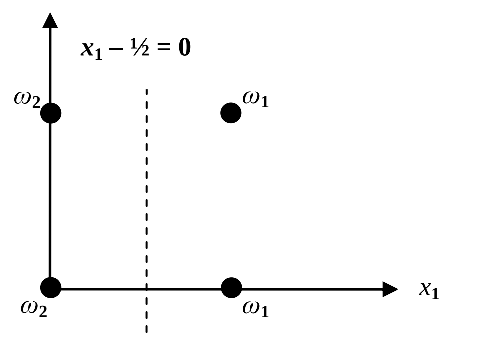
$$\boldsymbol{w}^T(1) = \boldsymbol{w}(0) - \frac{1}{1}\sum_{x\in Y} \delta_x \boldsymbol{x}$$

$$\boldsymbol{w}^T(1) = \begin{bmatrix}0\\1\\\frac{-1}{2}\end{bmatrix} - \frac{1}{1}\left((-1)\begin{bmatrix}1\\0\\1\end{bmatrix} + (+1)\begin{bmatrix}0\\1\\1\end{bmatrix}\right)$$

$$\boldsymbol{w}^T(1) = \begin{bmatrix}0\\1\\\frac{-1}{2}\end{bmatrix} + \begin{bmatrix}1\\0\\1\end{bmatrix} - \begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}1\\0\\\frac{-1}{2}\end{bmatrix}$$

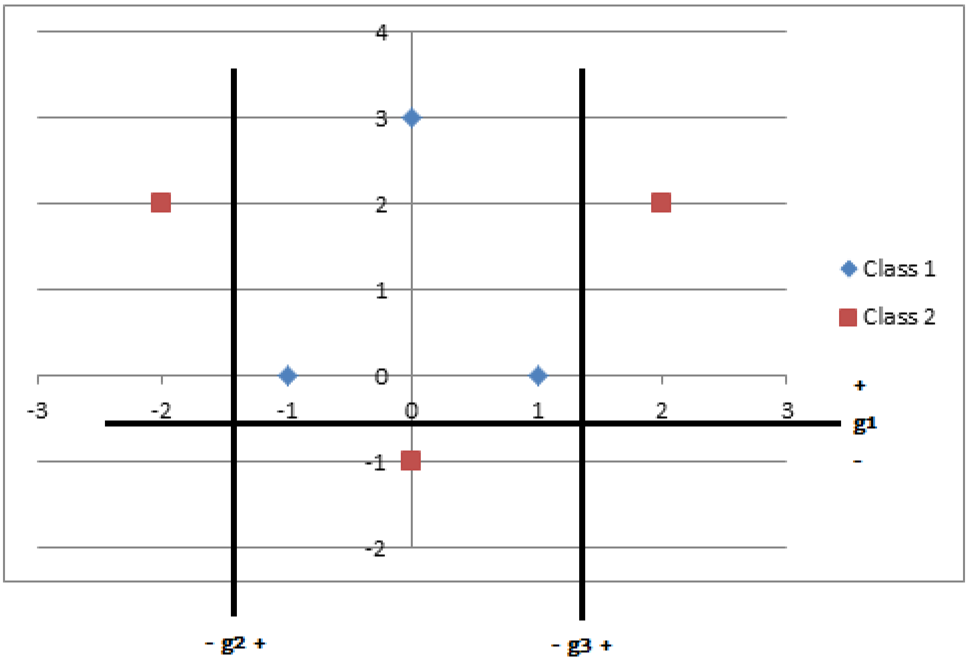
$$g_{t=1}(\boldsymbol{x}) = x_1 - \frac{1}{2}$$

The figure to the right shows that $g_{t=1}(\boldsymbol{x})$ correctly classifies all samples.



Q3. The following graph shows the feature values of 6 samples (three samples of Class 1 and three samples of Class 2). These classes are not linearly separable.

a) How many linear decision lines are needed to solve this problem?

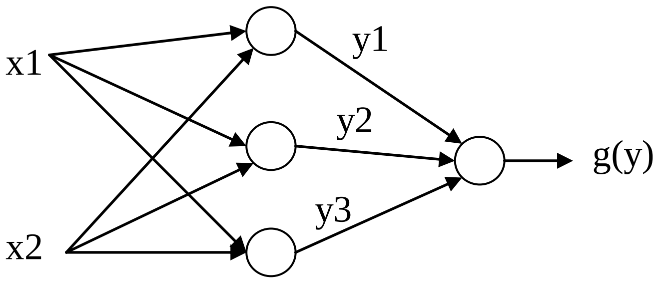


Select g1, g2, and g3 as shown in the figure.

Such that they satisfy the following table:

x1	x2	y1	y2	y3	Class
-1	0	1	1	0	1
1	0	1	1	0	1
0	3	1	1	0	1
-2	2	1	0	0	2
2	2	1	1	1	2
0	-1	0	1	0	2

b) Draw an appropriate two-layer perceptron network that can solve this problem.



c) Write the decision hyper-planes that each neuron of this network realizes.

$$g_1(\boldsymbol{x}) = x_2 + \frac{1}{2}$$

$$g_2(\boldsymbol{x}) = x_1 + \frac{3}{2}$$

$$g_1(\boldsymbol{x}) = x_1 - \frac{3}{2}$$

$$g(\boldsymbol{y}) = y_1 + y_2 - y_3 - 1.5$$

<Good Luck>