0907542 Patter Recognition (Spring 2012)
Midterm Exam Solution

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Instructions: Time **60** min. Closed books & notes. No calculators or mobile phones. No questions are allowed. **Bold** case is used for vectors and matrices. Show your work clearly. Every problem is for 6 marks.

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Q1. Using Bayes classification rule, find to what class the Point $x = 1$ belongs to? Given that:

$$P(\omega_1) = 0.75, \qquad P(\omega_2) = 0.25, \qquad p(1|\omega_1) = 0.4, \qquad p(1|\omega_2) = 0.8$$

$$P(\omega_1)\, p(1|\omega_1) \quad < (>) \quad P(\omega_2)\, p(1|\omega_2)$$

$$0.75 \times 0.4 \quad < (>) \quad 0.25 \times 0.8$$

$$0.3 \quad > \quad 0.2$$

Hence x belongs to ω_1

Q2. Recall that the decision hyper-plane in the sum of error squares method is found through the Equation $\hat{\boldsymbol{w}} = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{y}$. Write the Matlab code to find the decision hyper-plane for the following four samples:

$$\omega_1: \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \qquad \omega_2: \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}$$

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X = [0 0 1; 0 1 1; 1 0 1; 1 1 1];
y = [1 1 -1 -1]';
A = X' * X;
B = X' * y;
w = inv(A)*B;
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Q3. Consider a two-class (equi-probable), one-dimensional problem with samples distributed according to the Rayleigh pdf in each class, that is,

$$p(x|\omega_i) = \begin{Bmatrix} \frac{x}{\sigma_i^2} e^{\left(\frac{-x^2}{2\sigma_i^2}\right)} & x \geq 0 \\ 0 & x < 0 \end{Bmatrix}, \quad \sigma_1 = 1, \quad \sigma_2 = 2$$

Compute the threshold value x_0 for minimum risk if the loss matrix is:

$$\boldsymbol{L} = \begin{pmatrix} 0 & 0.5 \\ 1.0 & 0 \end{pmatrix}$$

The threshold x_0 occurs when

$$\lambda_{11} p(x|\omega_1) P(\omega_1) + \lambda_{21} p(x|\omega_2) P(\omega_2) = \lambda_{12} p(x|\omega_1) P(\omega_1) + \lambda_{22} p(x|\omega_2) P(\omega_2)$$

$$p(x|\omega_1) = p(x|\omega_2) \frac{P(\omega_2)}{P(\omega_1)} \frac{\lambda_{21} - \lambda_{22}}{\lambda_{12} - \lambda_{11}}$$

$$\frac{x}{\sigma_1^2} e^{\left(\frac{-x^2}{2\sigma_1^2}\right)} = \frac{x}{\sigma_2^2} e^{\left(\frac{-x^2}{2\sigma_2^2}\right)} \times 1 \times \frac{1.0 - 0}{0.5 - 0}$$

$$\frac{x}{1} e^{\left(\frac{-x^2}{2}\right)} = \frac{x}{4} e^{\left(\frac{-x^2}{2\times 4}\right)} \times 2$$

$$e^{\left(\frac{-x^2}{2}\right)} = \frac{1}{2} e^{\left(\frac{-x^2}{8}\right)}$$

Take the ln of both sides

$$\left(\frac{-x^2}{2}\right) = \ln\frac{1}{2} + \left(\frac{-x^2}{8}\right)$$

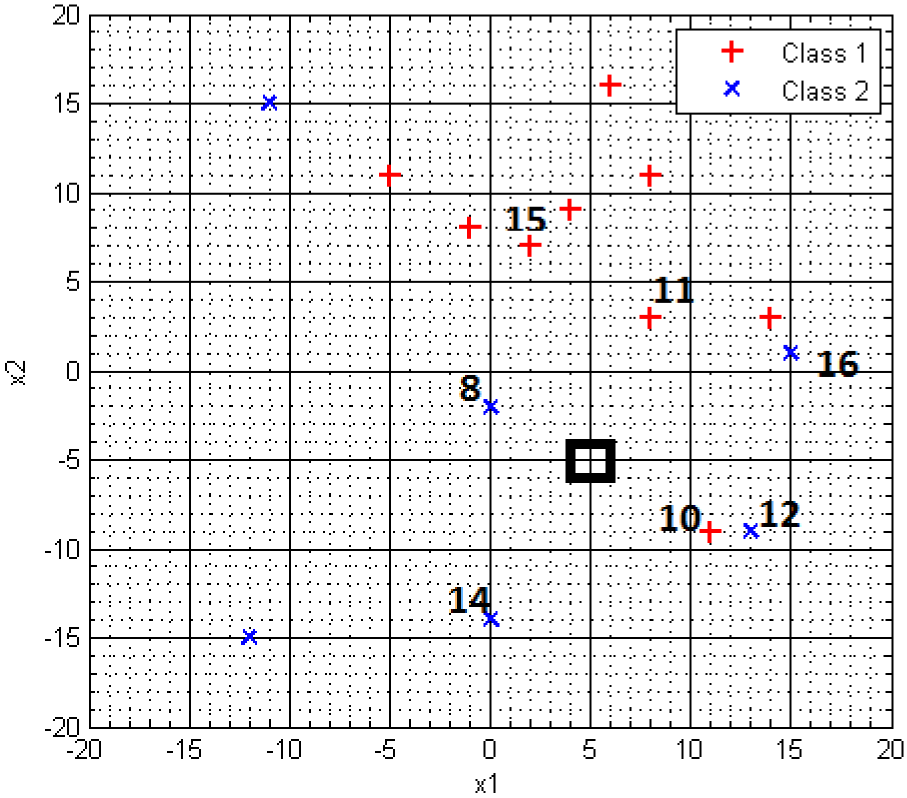
$$\frac{x^2}{2} - \frac{x^2}{8} = \ln 2$$

$$\frac{3x^2}{4} = 2\ln 2$$

$$x^2 = \frac{8}{3}\ln 2$$

$$x_0 = \sqrt{\frac{8}{3}\ln 2}$$

Q4. It is required to classify the point $\boldsymbol{x} = [5 \quad -5]^T$ using the k-nearest neighbor method. The known points are shown in the figure below and belong to Class 1 and Class 2. All feature values are integers. Classify this point for k=1, 3, and 5 and using the City Block distance metric.



The figure is marked with the distances of the nearest neighbors.

a) For k = 1 and distance metric is city block distance, the class is ____Class 2________

Nearest neighbor is 8(x)

b) For k = 3 and distance metric is city block distance, the class is ____Class 1________

Nearest 3 neighbors are 8(x), 10(+), 11(+)

c) For k = 5 and distance metric is city block distance, the class is ____Class 2________

Nearest 5 neighbors are 8(x), 10(+), 11(+), 12(x), 14(x)

Q5. In the sum of error squares method, the cost function used is

$$J(\boldsymbol{w}) = \sum_{i=1}^{N} (y_i - \boldsymbol{w}^T \boldsymbol{x}_i)^2$$

Show that the decision hyper-plane is $\hat{\boldsymbol{w}} = ((\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T)\boldsymbol{y}$, where $\boldsymbol{X} = \begin{bmatrix} \boldsymbol{x}_1^T \\ \boldsymbol{x}_2^T \\ \cdots \\ \boldsymbol{x}_N^T \end{bmatrix}, \quad \boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \\ \cdots \\ y_N \end{bmatrix}$

The minimum error occurs when:

$$\frac{\partial J(\boldsymbol{w})}{\partial \boldsymbol{w}} = \frac{\partial}{\partial \boldsymbol{w}} \sum_{i=1}^{N} (y_i - \boldsymbol{w}^T \boldsymbol{x}_i)^2 = 0$$

$$\sum_{i=1}^{N} 2\boldsymbol{x}_i (y_i - \boldsymbol{w}^T \boldsymbol{x}_i) = 0$$

$$\sum_{i=1}^{N} \boldsymbol{x}_i y_i - \boldsymbol{x}_i (\boldsymbol{w}^T \boldsymbol{x}_i) = 0$$

$$\sum_{i=1}^{N} \boldsymbol{x}_i (\boldsymbol{w}^T \boldsymbol{x}_i) = \sum_{i=1}^{N} \boldsymbol{x}_i y_i$$

$$\sum_{i=1}^{N} \boldsymbol{x}_i (\boldsymbol{x}_i^T \boldsymbol{w}) = \sum_{i=1}^{N} \boldsymbol{x}_i y_i$$

$$\sum_{i=1}^{N} \boldsymbol{x}_i^T \boldsymbol{x}_i \boldsymbol{w} = \sum_{i=1}^{N} \boldsymbol{x}_i y_i$$

$$\left(\sum_{i=1}^{N} \boldsymbol{x}_i^T \boldsymbol{x}_i \right) \boldsymbol{w} = \sum_{i=1}^{N} \boldsymbol{x}_i y_i$$

$$(\boldsymbol{X}^T \boldsymbol{X}) \boldsymbol{w} = \boldsymbol{X}^T \boldsymbol{y}$$

$$\boldsymbol{w} = (\boldsymbol{X}^T \boldsymbol{X})^{-1} \boldsymbol{X}^T \boldsymbol{y}$$

<Good Luck>