Homework 2 Solution

Problem 1 *(2 marks)*
Find the complement of the following function:
$X + Y(Z + (X + Z)')$

Change every variable to its complement and exchange ands and ors.

$$\begin{aligned} F' &= X'(Y' + (Z'(X + Z))) \\ &= X'(Y' + (Z'X + Z'Z)) \\ &= X'(Y' + XZ') \end{aligned}$$

Or applying De-Morgan theorem multiple times.

$$\begin{aligned} F' &= X'(Y(Z + (X + Z)'))' \\ &= X'(Y' + (Z + (X + Z)')') \\ &= X'(Y' + (Z'(X + Z)'')) \\ &= X'(Y' + (Z'(X + Z)'')) \\ &= X'(Y' + (Z'(X + Z))) \\ &= X'(Y' + (Z'X + Z'Z)) \\ &= X'(Y' + XZ') \end{aligned}$$

Problem 2 *(5 marks)*
For the Boolean Function F, as given in the following truth table:

X	Y	Z	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

(a) List its minterms
$F = \Sigma m(0, 1, 3, 5, 7)$

(b) List its maxterms
$F = \Pi M(2, 4, 6)$

(c) List the minterms of F'
$F = \Sigma m(2, 4, 6)$

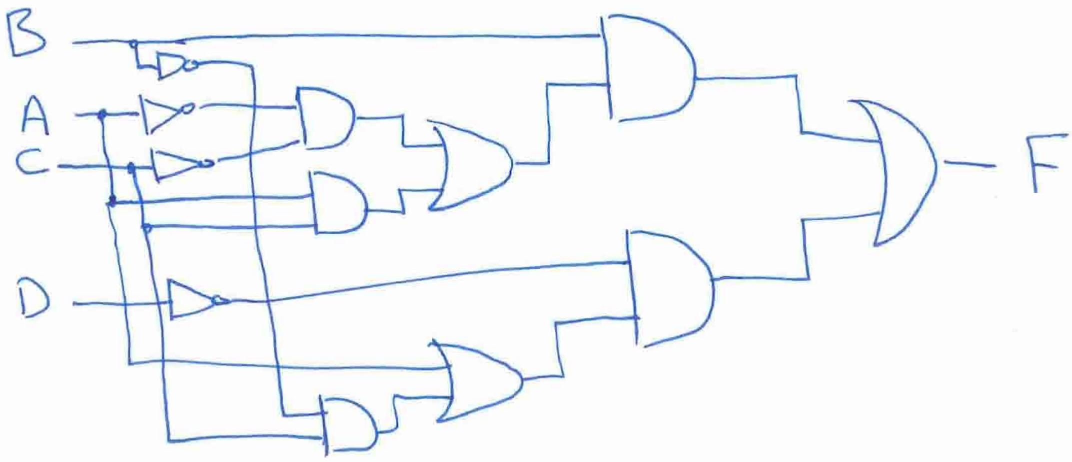
(d) Express F in sum-of-minterms algebraic form
$F = X'Y'Z' + X'Y'Z + X'YZ + XY'Z + XYZ$

(e) Express F in product-of-maxterms algebraic form
$F = (X + Y' + Z) . (X' + Y + Z) . (X' + Y' + Z)$

Problem 3 (2 marks)

Draw the logic diagram for the following Boolean expression. The diagram should correspond exactly to the equation. Assume the complements of the inputs are not available.

$$B(A'C' + AC) + D'(A + B'C)$$



Problem 4 *(6 marks)*

Optimize the following Boolean functions by means of Karnaugh map:

(a) F(X, Y, Z) = Σm(0, 2, 3, 4, 6)

X \ YZ	00	01	11	10
0	1		1	1
1	1			1

F = Z' + X'Y

(b) F(W, X, Y, Z) = Σm(0, 2, 5, 6, 8, 10, 13, 14, 15)

WX \ YZ	00	01	11	10
00	1			1
01		1		1
11		1	1	1
10	1			1

F = X'Z' + YZ' + XY'Z + WXZ

Or

F = X'Z' + YZ' + XY'Z + WXY

(c) F(W, X, Y, Z) = Σm(2, 4, 9, 12, 15), with the don't-care conditions d(W, X, Y, Z) = Σm(3, 5, 6, 13)

WX \ YZ	00	01	11	10
00			X	1
01	1	X		X
11	1	X	1	
10		1		

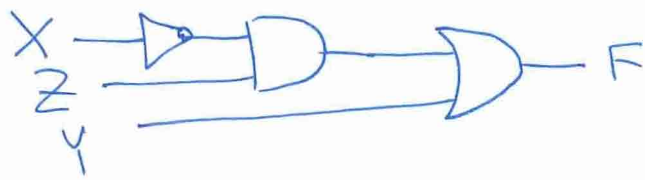
F = XY' + WY'Z + WXZ + W'X'Y

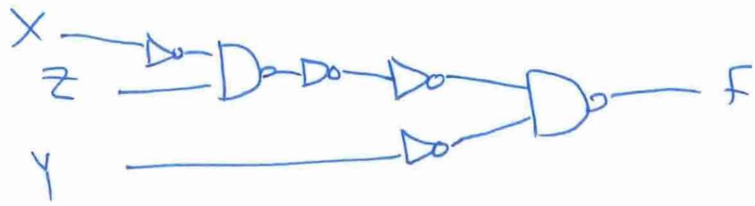
Or

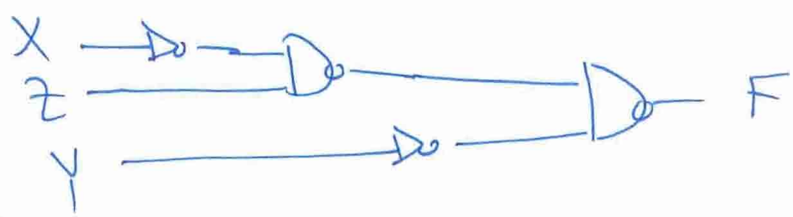
F = XY' + WY'Z + WXZ + W'YZ'

Problem 5 (2 marks)

Implement the following Boolean function using NAND gates only: F(X, Y, Z) = X'Z + Y



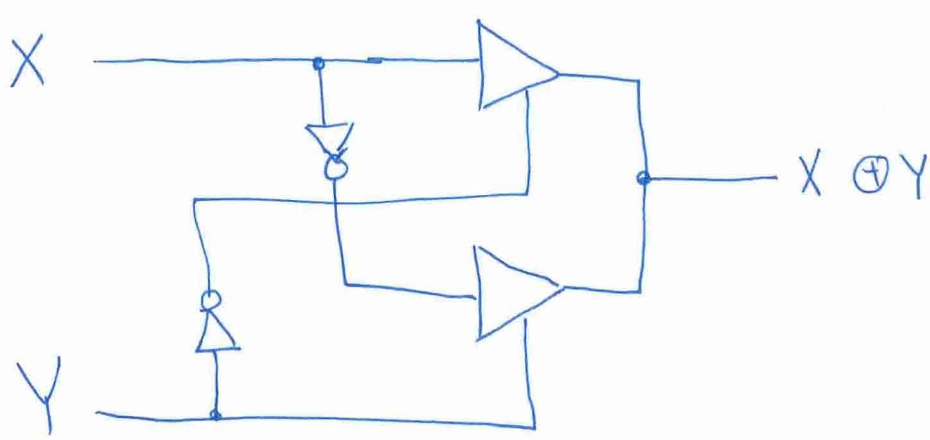




$$\overline{\overline{\bar{X}Z}\cdot\bar{Y}} = \bar{X}Z + Y$$

Problem 6 (2 marks)

Construct an exclusive-OR gate by interconnecting two three-state buffers and two inverters.



Problem 7 *(6 marks)*
Obtain the truth table of the following functions, and express each function in sum-of-minterms and product-of-maxterms form:

(a) (AB + C)(B + AC)

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

F = Σm(3, 5, 6, 7)
F = ΠM(0, 1, 2, 4)

(b) ABC' + ABD' + ABD + CD'

A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

F = Σm(2, 6, 10, 12, 13, 14, 15)
F = ΠM(0, 1, 3, 4, 5, 7, 8, 9, 11)

Problem 8 *(4 marks)*

(a) Convert the following Boolean expression into SOP then into SOM: $X' + X(X+Y')(Y+Z')$

$F = X' + X(X+Y')(Y+Z')$
$= X' + (XX+XY')(Y+Z')$
$= X' + (X+XY')(Y+Z')$
$= X' + (X)(Y+Z')$
$= X' + XY + XZ'$ *(SOP form)*

$= X'(Y'Z'+Y'Z+YZ'+YZ) + XY(Z'+Z) + XZ'(Y'+Y)$ *Adding missing minterms*
$= X'Y'Z'+Z'Y'Z+ZYZ'+X'YZ + XYZ'+XYZ + XY'Z'+XYZ'$
$= X'Y'Z'+Z'Y'Z+ZYZ'+X'YZ + XYZ'+XYZ + XY'Z'$ *Removing duplicates*
$= X'Y'Z'+Z'Y'Z+ZYZ'+X'YZ + XY'Z' + XYZ'+XYZ$ *(SOM form)*

(b) Convert the following Boolean expression into POS then into POM: $(AB+C)(B+AC')$

$F = (AB+C)(B+AC')$
$= (A+C)(B+C)\ (B+A)(B+C')$
$= (A+C)\ (B+C)\ (A+B)\ (B+C')$ *(POS form)*

Adding missing maxterms
$= (A+B'+C)(A+B+C)\ (A'+B+C)(A+B+C)\ (A+B+C')(A+B+C)\ (A'+B+C')(A'+B+C')$
Removing duplicates
$= (A+B'+C)(A+B+C)\ (A'+B+C)\ (A+B+C')\ (A'+B+C')$
Rearranging
$= (A+B+C)\ (A+B+C')\ (A+B'+C)\ (A'+B+C)\ (A'+B+C')$ *(POM form)*